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# THE INFLUENCE OF ATMOSPHERIC PRESSURE UPON THE FORCED THERMAL CONVECTION FROM SMALL ELECTRICALLY HEATED PLATINUM WIRES.

BY A. E. KENNELLY AND H. S. SANBORN.

(Read April 24, 1914.)

## OBJECT OF ENQUIRY.

This paper describes the process and results of a research made at Harvard University in 1911, to determine the effect of change in atmospheric pressure on forced thermal convection from thin platinum wires. By forced thermal convection is meant the carrying away of heat from the surface of a wire by wind-motion, *i. e.*, by a rapid transverse movement of the wire through the surrounding air. This wind motion through the air dissipates the heat from the wire convectively. The rate of thermal convection depends upon the length and diameter of the wire, its surface condition, the temperature elevation of the wire above the air, the velocity of the motion, and the pressure of the air. The object of the enquiry was to determine the effect of the last-named variable—variation of atmospheric pressure—upon the thermal dissipation, the other quantities being kept constant.

## HISTORY OF THE ENQUIRY.

The research here described was the outcome of an earlier investigation on "The Convection of Heat from Small Copper Wires," by Messrs. A. E. Kennelly, C. A. Wright and J. S. Van Bylevelt, presented at the Frontenac Convention of the American Institute of Electrical Engineers, June 28, 1909, and published at p. 363, Vol. XXVIII., part I., of the *Transactions* for that year. In that research, the forced convection of heat from a thin copper wire, electrically heated to a constant temperature, *i. e.*, maintained at a constant

electric resistance, was discovered to vary as the square root of the wind velocity, which was measured by the speed of the moving wire through otherwise tranquil air. In other words, it was discovered experimentally that in order to dissipate double the power from the wire, at constant resistance and temperature-elevation, it was necessary to quadruple the speed of the wire through the air. This relation was found to hold, within observation errors, for several different sizes of thin copper wire, and for various temperature elevations, between wind-velocities of 2 and 20 meters per second. Below 2 meters per second, the relation deviated towards the case of free convection from a hot wire at rest. That is, at low wind velocities, empirical corrections became necessary for the free convection which naturally occurs from a wire at rest, or moving at zero speed through the air. The possible application of the square-root law of wind cooling to anemometry was also pointed out.

After the results were published in 1909, our attention was drawn to papers by Professor Boussinesq in the *Comptes Rendus* for 1901, Vol. 133, p. 257, and the *Journal de Mathématiques*, 6th Series, Vol. 1, 1905, in which is given the theory of the convection of heat by a stream of liquid from the surface of a cylindrical rod, placed at right angles to the stream. The liquid is assumed to be incompressible and devoid of viscosity. The formula arrived at by Boussinesq, as given by Russell, is:

$$H = 8\theta \sqrt{\frac{s\sigma kVa}{\pi}},$$

where  $H$  is the heat carried off convectively per second from unit length of cylinder.

$s$  is the specific heat of the liquid.

$\sigma$  is the density of the liquid.

$k$  the thermal conductivity of the liquid.

$V$  is the velocity of the liquid.

$a$  the radius of the cylinder.

$\theta$  the temperature elevation of the cylinder.

This means that the linear forced convection, or ergs per second per cm. of the cylinder, is proportional to its temperature-elevation above

the liquid, and to the square root of the specific heat, the thermal conductivity, the wind velocity, the fluid density and the wire radius.

Dr. Alexander Russell communicated an important paper on the theory of the subject to the Physical Society of London in July 1910,<sup>1</sup> developing and extending Boussinesq's formula.

Professor J. T. Morris has recently successfully applied the square-root law of forced-convection velocity to the measurement of wind-velocities, using an ingenious form of Wheatstone bridge for this purpose. His observations were communicated to Section G of the British Association in 1912<sup>2</sup> and also to "Engineering"<sup>3</sup> in 1913. His results have confirmed the application of the law for wires of various metals up to diameters of 0.3 mm.

The papers and deductions of Boussinesq were not known to us at the time we presented our former paper in 1909; but since the square-root law of velocity arrived at theoretically by Boussinesq in 1901-1905, for an incompressible non-viscous liquid, has been found to hold within errors of observation for ordinary air, it became desirable to ascertain whether the linear forced convection of air varied as the square root of the air pressure, as suggested by Boussinesq's formula.

#### METHOD OF MEASUREMENT EMPLOYED.

The method followed and the apparatus used were respectively the same as those described in the A. I. E. E. paper of 1909, above referred to. A short length of the thin wire to be tested was held in a fork, and was driven by an electric motor at successively varied speeds in a large steel tank, the atmospheric pressure within which was kept constant in each series of tests; but was different in different series.

#### TEST WIRE.

The wire used in all of the tests here described was of good commercial platinum, No. 36 B. & S. gauge, with a mean diameter of

<sup>1</sup> *Proc. Physical Society*, 1910, Vol. XXII., also *Phil. Mag.*, October, 1910.

<sup>2</sup> Prof. J. T. Morris, "The Electrical Measurement of Wind Velocity," *The Electrician*, Oct. 4, 1912, pp. 1056-1059.

<sup>3</sup> J. T. Morris, "Distribution of Wind Velocity about a Circular Rod," *Engineering*, Vol. 96, pp. 178-181, Aug. 8, 1913.

0.114 mm. (0.0045 inch). In the tests of 1909, copper wires were used. The advantage of copper is that its resistivity temperature-coefficient is relatively large, and is fairly reliable. On the other hand, hot copper wires oxidize superficially when driven through the air, and are therefore subject to variation in convective dissipation, owing to this change of surface condition. As the test-wire in the new measurements had to be driven inside a steel tank, with only occasional inspections, it was decided to employ platinum, instead of copper; although the resistivity temperature-coefficient of the platinum was but little more than half that of copper; so that the resistance of such a platinum wire is not so sensitive to changes of temperature as a copper wire. Consequently, greater care was needed in the electrical measurements of resistance in the platinum test-wire, in order to determine the temperature elevation.

A measurement of the temperature-coefficient of resistivity of the platinum wire used was made by immersing 5.5 meters of it on a reel in an oil-bath, and measuring the resistance at twelve different temperatures between 0° C. and 100° C. As shown in Fig. 6, the results obtained lie close to the straight line:

$$\rho_t = \rho_0(1 + 0.002575t) \quad \text{absohm-cm (1)}$$

where  $\rho_t$  is the resistivity at  $t^\circ$  C. (absohm-cm.), and  $\rho_0$  is the resistivity at 0° C. (absohm-cm.).

The particulars concerning the test wire are given in the accompanying Table:

TABLE I.  
TEST-WIRE DIMENSIONS AND DATA.

Mean Diameter.		Cross-Sectional Area, sq. cm.	Linear Surface, sq. cm./cm.	Linear Mass, gm./cm.	Linear Res. at 0° C., absohms/cm.	Resistivity at 0° C., absohm-cm.	Temp. Coeff. of Resistivity 0° C.
mm.	Inch.						
0.114	0.0045	1.02 x 10 <sup>-4</sup>	0.0358	2.415 x 10 <sup>-3</sup>	1.28 x 10 <sup>8</sup>	1.306 x 10 <sup>4</sup>	0.002575

#### TEST-WIRE HOLDER.

The test-wire was held in a fork or frame, mounted on the shaft of the driving motor. The fork is indicated in Fig. 1. It is counterpoised by the sliding weight  $f$ . The test-wire is shown at  $b$ , held

straight and fairly tight, by the elasticity of the brass strip prongs  $aa'$ , aided by the tension-screws  $gg^1$ . Current is steadily supplied to the test-wire through slip-rings  $cc'$ , on which rest stationary copper gauze brushes of square cross-section 0.64 cm. ( $\frac{1}{4}$  inch) on each

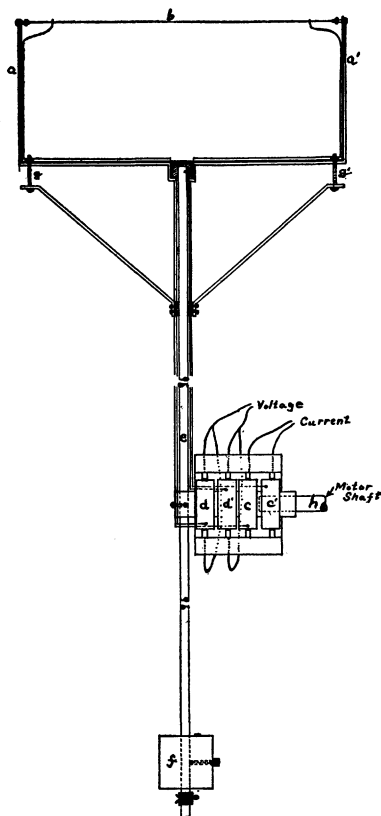


FIG. 1. Details of Rotatable Fork Supporting the Test Wire.

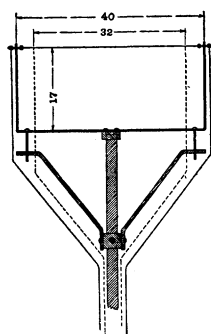


FIG. 2. Details of Fork. Dimensions in Centimeters.

edge. The details of the fork dimensions are shown in Fig. 2. At two points along the test-wire 32 cm. apart, pressure wires are soldered to the test-wire. These pressure wires are of platinum, of the same size as the test-wire. They connect with insulated copper wires fastened to the sides of the fork, and terminate in the slip-rings  $dd'$  carried by the motor shaft, on which rest two pairs of

stationary gauze brushes. The electrical connections are as shown in Fig. 3. The source of e.m.f. was a storage battery. The regulating resistance  $RR$  was so adjusted that the ratio of the p.d. between pressure-wires, to the current strength, was equal to a predetermined resistance. That is, the current in the test-wire was gradually increased until the ratio of the reading of the voltmeter  $V$  to that of the ammeter  $A$ , was found, by slide-rule, to give the correct resistance sought to be maintained in the test-wire at all wind speeds.

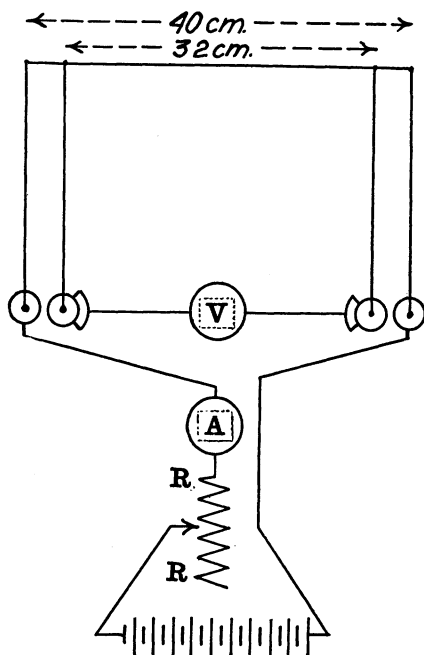


FIG. 3. Diagram of Electrical Connections.

As the driving speed increased, the current supplied to the wire had to be increased, in order to maintain this ratio  $V/A$ . When, on the contrary, the driving motor was brought to rest, the current in the test-wire had to be reduced to a relatively small value, in order to reproduce the ratio.

The fork was mounted on the shaft of a  $\frac{1}{2}$ -HP. 115-volt direct-current shunt motor, arranged to run at adjustable speeds. The

wind speed of the test wire in cm. per sec. was taken as  $2\pi \times \text{cm. fork radius} \times \text{speed of motor in r.p.s.}$  At the other end of the motor-shaft was coupled a small magneto-generator for indicating, by its e.m.f., the speed of rotation. The fork-motor-magneto mechanism is illustrated in Fig. 4, supported on a wooden frame intended to be held in place inside the pressure tank, which is shown with the manhole open.

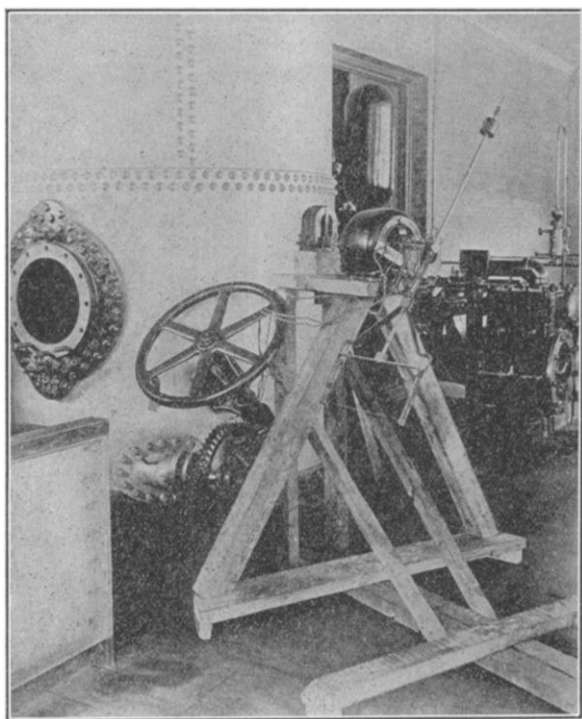


FIG. 4. Photograph of Fork, Driving-Motor, Magneto Speed-Indicator and Pressure Tank.

#### PRESSURE TANK.

The pressure tank in which the motor and fork were supported was a vertical steel cylinder of  $\frac{1}{2}$ " (1.25 cm.) steel plates, riveted. Figure 5 shows the dimensions of the tank, and also the position within it occupied by the motor and fork. The radius of the fork



to the test-wire; *i. e.*, the distance of the test-wire from the motor-shaft axis was 58.5 cm. (23"); and the radius of the pressure tank was 76 cm. (30"), leaving a clearance between the rotating wire and the tank wall of 17.5 cm. (7"). A larger pressure tank, allowing

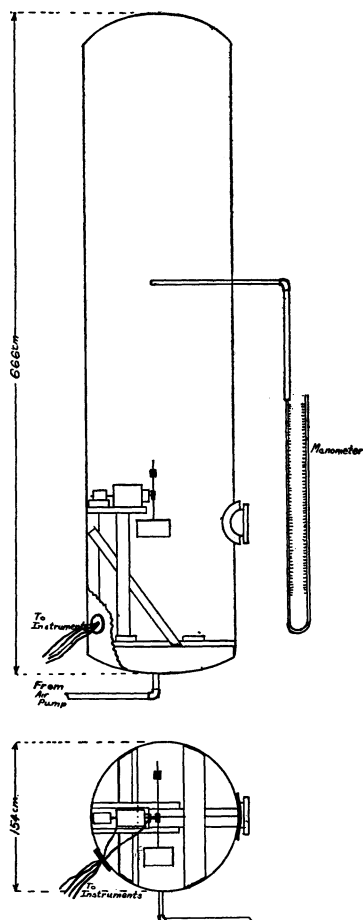


FIG. 5. Elevation and Plan of Pressure Tank showing position occupied by the Test-Wire.

more space and clearance for the revolving test-wire, would have been preferable; but the arrangement was the best that could be made with the apparatus at hand. The results obtained at any

single air-pressure were not so good as those obtained at normal atmospheric pressure in the open air outside the tank, with a larger fork radius, and free air-space. That is, the curves of linear convection against wind-velocity, on logarithm paper, showed more tendency to deviate from a straight line, in these tank tests than in open-room tests, both at low speeds and at high speeds. These deviations might perhaps be explained by air-churnings in the tank, due to the motion of the fork and wire in a somewhat confined space.

The insulated wires leading to motor, magneto, and test-wire, were brought out through holes in a wooden plug bolted air-tight over a manhole.

The speed of rotation of the motor inside the tank was measured in two independent ways; namely (1) by the e.m.f. of the little magneto-generator coupled to the motor, (2) by a contact made

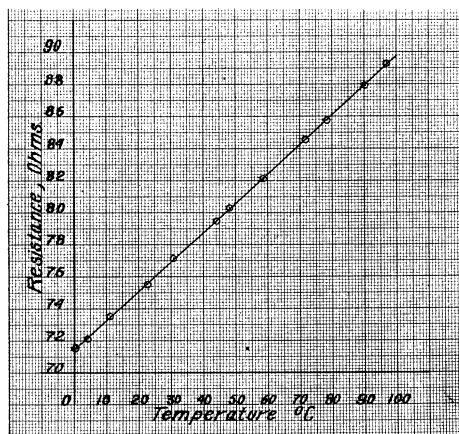


FIG. 6. Resistance in oil of 550 cm. of the Platinum Test Wire at different temperatures between  $0^{\circ}$  C. and  $97^{\circ}$  C. in order to determine the resistivity temperature-coefficient.

through a wire on the motor shaft once in each revolution, which gave a click in a telephone. The speed calibration of the magneto and its voltmeter could thus be checked, from time to time, by counting the telephonic clicks in one minute.

The pressure of the air in the tank was controlled by pumps connected with the tank. The tank was fairly tight and ordinarily held its pressure steadily during a test. A large glass U-tube containing mercury was connected with the tank. The difference of level between the mercury in the two arms of the U, corrected for temperature, gave the difference of pressure between the air inside and outside the tank. The absolute pressure of the air in the tank was thus the sum of the U-tube pressure and the corrected barometer pressure outside. This absolute pressure was expressed in "bars" or C.G.S. units (dynes per sq. cm.), by allowing 75.009 cm. of mercury to 1 megabar or  $10^6$  bars.<sup>4</sup>

#### HOT-WIRE TEMPERATURES.

Two hot resistances were selected for the 32 cm. length of test-wire in different series of tests; namely one at 8.44 ohms, and the other at 10.0 ohms, corresponding to temperatures of  $410^{\circ}$  C. and  $558^{\circ}$  C. respectively, by extrapolation from the calibration test between  $0^{\circ}$  C. and  $100^{\circ}$  C. indicated in Fig. 6. These temperatures are therefore inferred by resistance. If the temperatures of the wire actually differed from the above inferred values, the values of linear convection here deduced would be correspondingly changed; but the comparative results would be unchanged. So far as the main subjects of enquiry are concerned, it is sufficient that the wire returns to one and the same definite temperature when heated electrically to one and the same resistance. With the air-temperature in the tank in the neighborhood of  $20^{\circ}$  C., the inferred temperature-elevation of the test-wire by resistance was  $390^{\circ}$  C. and  $538^{\circ}$  C. About ten series of speed-measurements were made at each of these elevations, with different air-pressures.

The following table gives one series of tests as an example.

<sup>4</sup> "Les Récents Progrès du Système Métrique," Paris, Gauthier-Villars, 1907, pp. 30-31.

TABLE II.

SERIES OF MEASUREMENTS ON FEBRUARY 16, 1911. Observers A. E. K. and H. S. S. Pressure in tank 75.2 cm. Hg above that in room. Barometer 778 mm. at 14.5° C. Temp. air in tank 18.5° C. Mean absolute pressure in tank 2.04 megabars. Res. of test platinum wire between pressure wires kept at 8.44 ohms. Inferred temp. elevation 391.5° C. R. P. M. of driving motor =  $1.14 \times$  magneto voltage.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Magneto Volts.	Wind Velocity $v$ cm./sec.	P. D. on 32 cm. $E$ abvolts.	Current in Wire $I$ abs. amp.	Linear Dissipation $P_e = E \cdot I / 32$ abwatts/cm.	Linear Dissipation per ° C. $P_e / \theta$ abwatts/cm. deg. C.	$\frac{P_e}{\theta \sqrt{v}}$	$\frac{P_e}{\theta \sqrt{v} + 30}$
		$\times 10^8$		$\times 10^7$	$\times 10^4$		
101	705.5	23.5	0.279	2.051	5.24	1973	1930
126	880.5	24.9	0.295	2.294	5.86	1975	1942
149	1041	26.0	0.308	2.501	6.390	1981	1952
174	1216	26.95	0.320	2.695	6.880	1975	1948
203	1418	27.8	0.330	2.873	7.340	1949	1928
225	1572	28.55	0.339	3.030	7.740	1954	1929
250	1747	29.3	0.348	3.194	8.160	1954	1935
274	1914	30.0	0.356	3.343	8.539	1952	1935
303	2117	30.6	0.363	3.476	8.879	1930	1915
310	2166	30.9	0.367	3.553	9.075	1950	1938
100	699	23.35	0.277	2.023	5.167	1954	1912
74	517	21.75	0.258	1.754	4.480	1970	1916
43	300.4	19.55	0.232	1.420	3.628	2092	1995
0	0	10.5	0.125	0.412	1.050	$\infty$	1917

Column I gives the voltage generated by the magneto on the driving-motor shaft. From these readings, the speed of the test-wire through the air in the tank; or the "wind-velocity"  $v$  of Column II is directly derived in cm. per sec. Column III gives, at each steady speed, the P.D. between pressure wires in C.G.S. magnetic absolute units or "abvolts." Column IV gives the corresponding current strength in C.G.S. magnetic absolute units or "absamperes." The ratio  $E/I$  is approximately constant at  $8.44 \times 10^9$  absohms, or 8.44 ohms. Column V gives the linear dissipation of heat from the wire, or the abwatts (ergs per second) per cm. of wire length. Column VI gives this linear dissipation per deg. Cent. of inferred temperature elevation. It will be seen that this varies from  $1.05 \times 10^4$  abwatts per cm. and ° C. at standstill, up to  $9.075 \times 10^4$  at 2,166 cm. per sec. Column VII gives the entries of VI divided by  $\sqrt{v}$  giving a value nearly uniform about 1,960, until the velocity  $v$  falls to 300 cm. per sec. At standstill, of course, owing

to free convection, the value becomes infinite. If, however, we add 30 cm. per sec. to all the wind velocities  $v$ , to correct empirically for free convection as described in the paper of 1909, we obtain the values given in the last column VIII, which do not differ greatly from 1,930 abwatts per cm. ° C. and  $\sqrt{v}$ , at all speeds in the table.

It will be observed that no correction is made for loss of heat by radiation from the test-wire. That is, the linear dissipations in column VI are treated as though entirely due to convection. In our paper of 1909, a correction was attempted for radiation, on the basis

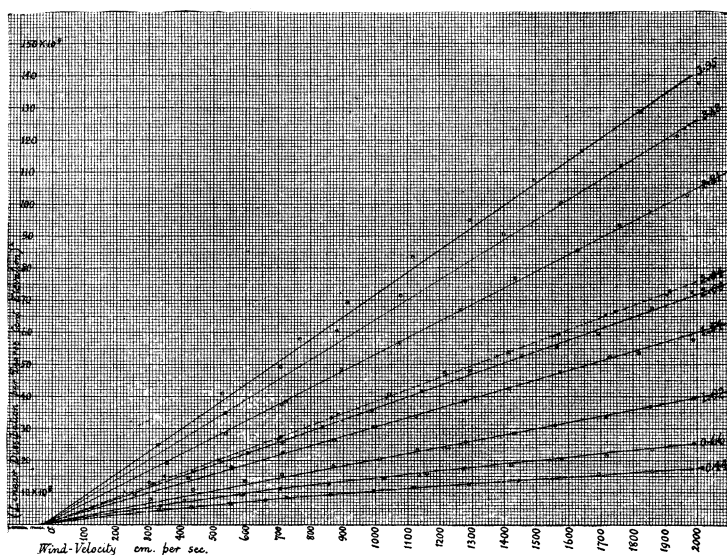


FIG. 7. Curves of  $\left(\frac{P}{\theta}\right)^2$  against  $v$ , for  $\theta = 390^\circ$ .

of Stefan's formula. Since, however, it has been pointed out by Dr. Langmuir<sup>5</sup> that the radiation from platinum according to Hagen and Ruben's formula is only a small fraction of that from a "black body," or perfectly non-reflecting radiator, at the same temperature, the radiation corrections in the case of Table II are nearly all less than 1 per cent. of the dissipation, and it has therefore been omitted throughout.

<sup>5</sup> "The Convection and Conduction of Heat in Gases," by Irving Langmuir, *Proc. Am. Inst. El. Engrs.*, June 25, 1912.

The relation found in our 1909 paper was that

$$P_c = k\theta\sqrt{v} \quad \text{abwatts per cm.} \quad (2)$$

where  $P_c$  is the linear convection from the hot wire in abwatts per cm.,  $\theta$  the temperature elevation of the wire, in degrees Centigrade,  $v$  the wind-velocity or speed of transverse motion of the wire through the air in cm. per second, and  $k$  a constant depending, among other things, on the size and surface-condition of the wire. This formula was found to hold well between the wind velocities  $v=200$  and  $v=2000$  cm./sec. (7.2 and 72 km/hr. or 4.47 and 44.7 statute miles/hr.); but not to hold below  $v=200$ , unless 30 cm. per sec. were added as an empirical correction to all speeds to take free convection into approximate account. This empirical correction, applying fairly well, gave:

$$P_c = k\theta\sqrt{v + v_0} \quad \text{abwatts per cm.} \quad (2a)$$

where  $v_0$  is a virtual velocity of free convection approximating 30 cm. per second.

The relation indicated in (2) can be presented graphically by straight lines on logarithm-paper, but Professor Morris has employed the corresponding relation:

$$\left(\frac{P_c}{\theta}\right)^2 = k^2 v \quad \left(\frac{\text{abwatts per cm.}}{\text{deg. C.}}\right)^2 \quad (3)$$

That is he plots the square of the observed linear convection per degree C., against the wind velocity, thus producing a straight line, if either (2) or (2a) applies. The procedure is followed in Figs. 7 and 8. Thus, taking Fig. 7, the broken straight line marked 2.04 corresponds to the results in an air-pressure of 2.04 megabars, and the observations in Table I appear on or near this line as small circles. Nine different series are indicated in Fig. 7 at pressures of 0.44, 0.66, 1.02, 1.54, 2.00, 2.04, 2.81, 3.48 and 3.95 megabars respectively, the first two corresponding to vacua, 1.02 to normal atmospheric pressure, and the six others to extra pressure in the tank.

It will be seen that the two lowest curves—vacua—deviate distinctly from straight lines. The remainder are drawn as straight lines, and the observations conform to them fairly well, except at the

two highest pressures 3.48 and 3.95 megabars. This means that equations (2) and (3) held satisfactorily from 1 to 2.8 megabars, but did not hold so well outside those limits of pressure.

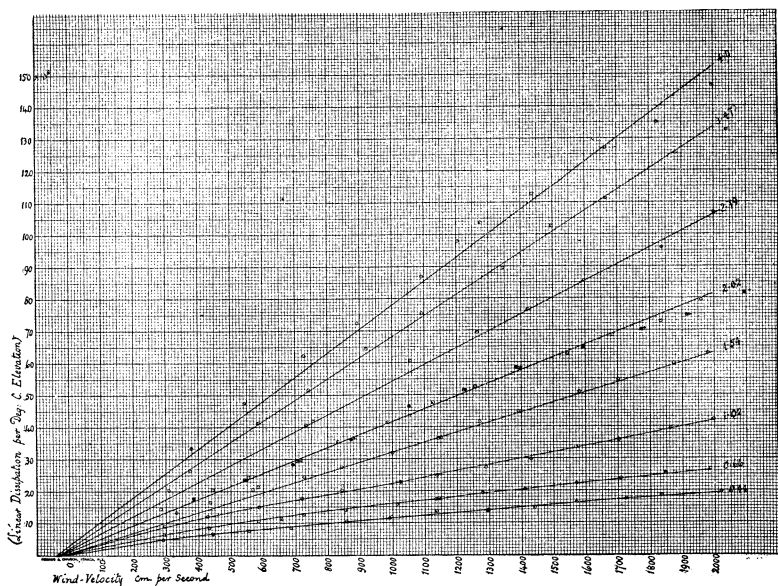


FIG. 8. Curves of  $\left(\frac{P}{\theta}\right)^2$  against  $v$ , for  $\theta = 538^\circ$ .

In order to eliminate, as far as possible, any disturbing influence on the forced convection in these tests due to the presence of water-vapor in the air contained in the tank, calcium chloride was kept in the tank. The measurements were all made between January 23 and February 17, 1911, at a time of the year when the air in Cambridge is ordinarily relatively free from moisture. In order to find whether moisture in the air had any considerable effect on the forced convection of heat from the test-wire, one test was repeated at one air-pressure (2 megabars) at each of the two temperature-elevations, first with the air after it had been exposed to the calcium chloride, and second with the calcium chloride removed and a dish of water set over night in its place. The actual difference in the humidity of the air in the tank was not measured, but it was supposed that there

would be a marked difference. It will be seen in Fig. 7 that the small circles on the 2.04 megabar line, representing the dried air test, fairly coincide with the small crosses representing the air test in the presence of water. The same is true for the 2.02 megabar line of Fig. 8. Consequently, the effect of moisture in the heat convection of moving air has not yet been determined from our tests, although it would seem reasonable that in view of the very appreciable known thermal capacity of aqueous vapor, the effect of moisture might have been apparent.

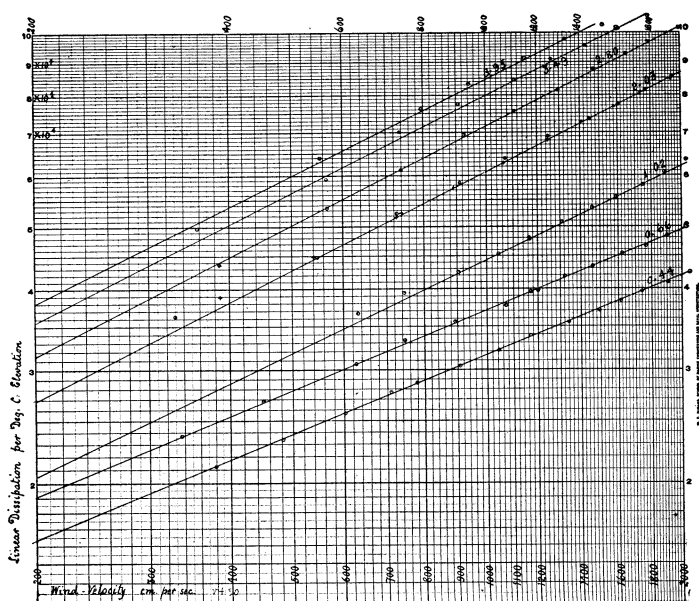


FIG. 9. Graphs of  $\left(\frac{P}{\theta}\right)$  against  $v$  at different pressures for  $\theta = 390^\circ \text{ C.}$   
logarithm paper.

It will be seen that the observations in Figs. 7 and 8 indicate a relation:

$$\left(\frac{P_c}{\theta}\right)^2 = k^2(v + v_0) \quad \left(\frac{\text{abwatts per cm.}}{\text{deg. C.}}\right)^2 \quad (4)$$

where  $v_0$  is a velocity in the neighborhood of 30 cm. per second, which may be assumed as the empirical correction due to free convection



from the test-wire when held stationary in the air. Professor Morris's method of graphic representation has the advantage that it indicates directly the magnitude of the empirical correction  $v_0$ . If we take  $v_0 = 30$ , we have from (4)

$$\frac{P_c}{\theta} = k\sqrt{v + 30} \quad \frac{\text{abwatts per cm.}}{^\circ\text{C.}} \quad (5)$$

Figs. 9 and 10 give the graphs of  $P_c/\theta$  on logarithm-paper, for the various sets of observations. It will be seen the observations lie not far from straight lines. These lines have a gradient of 1:2, or correspond to a square-root law, or exponent of 1/2 as in (5); except for the vacua (0.44 and 0.66 megabar), where the gradient is approximately 4:10; or would more nearly indicate a relation

$$\frac{P_c}{\theta} = k(v + 30)^{0.4} \quad \frac{\text{abwatts per cm.}}{^\circ\text{C.}} \quad (6)$$

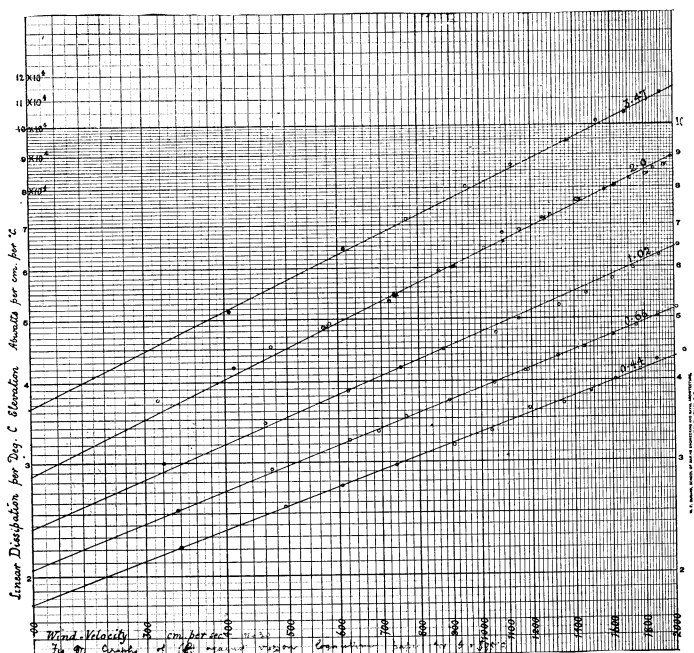


FIG. 10. Graphs of  $\left(\frac{P}{\theta}\right)$  against  $v + 30$  on logarithm paper for  $\theta = 538^\circ\text{C.}$

Considering the results indicated in Figs. 7 and 8, it appears that the slopes of the various straight lines are nearly proportional to the atmospheric pressure. This means that, at least to a first approximation:

$$\left(\frac{P_c}{\theta}\right)^2 = pk'(v + v_0) \quad \left(\frac{\text{abwatts per cm.}}{\text{deg. C.}}\right)^2 \quad (7)$$

where  $pk' = k^2$

Consequently, when in Professor Morris's diagram, the atmospheric pressure  $p$  changes, the ordinates are increased in like proportion. We have then

$$P_c = \theta \sqrt{pk'(v + v_0)} \quad \frac{\text{abwatts per cm.}}{\text{deg. C.}} \quad (8)$$

so that the linear convection is nearly proportional not only to the square root of the velocity, but also to the square root of the atmospheric pressure. This agrees with Boussinesq's formula (o) when it is remembered that the air-density  $\sigma$  of the medium is proportional to the pressure  $p$ . The remaining constant  $k'$  involves, among other things, the diameter of the wire. According to Boussinesq's formula (o), the constant  $k'$  should be proportional to the square root of the wire diameter. A special investigation should be directed to this question: but the measurements recorded in our earlier paper of 1909 seem to indicate a higher ratio than the square root.

TABLE III.

VALUES OF  $k'$  IN THE EXPRESSION  $P_c/\theta = \sqrt{k'p(v + v_0)}$  derived from the series of observations at different atmospheric pressures  $p$ , at the velocity  $(v + v_0) = 1,000$  cm. per sec.

$\theta = 390^\circ.$		$\theta = 538^\circ.$	
$p$ Bars.	$k'$	$p$ Bars.	$k'$
$3.95 \times 10^6$	1.87	$4.0 \times 10^6$	1.98
3.48 "	1.87	3.47 "	1.96
2.81 "	1.85	2.79 "	1.90
2.04 "	1.83	2.02 "	2.00
2.00 "	1.76		
1.54 "	1.915	1.54 "	2.01
1.02 "	1.96	1.02 "	2.16
0.66 "	2.15	0.66 "	2.39
0.44 "	2.14	0.44 "	2.50

Table III shows the value of  $k'$  in formulas (7) and (8) for the various series of measurements appearing in this report at the velocity  $v=1,000$  cm. per second. It will be seen that  $k'$  varies between 1.76 and 2.50, with a mean value near 2.0.

#### ANEMOMETER MEASUREMENTS.

A wind-velocity measuring apparatus, or electric anemometer, was constructed of the same thin platinum wire as that used in the preceding tests (0.114 mm. diameter). A length of 25 cm. of this

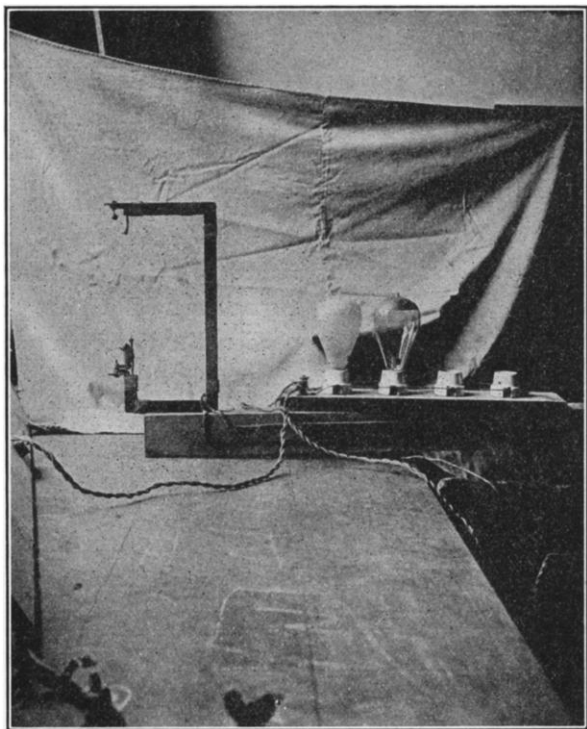


FIG. II. Experimental Anemometer.

wire (10 in.) was supported vertically, between insulated clips, in a steel  $G$  frame shown in Fig. II. Pressure taps, of the same size platinum wire, were soldered on to the vertical test-wire, at a distance

of 15 cm. (5.9 in.) apart. The vertical test-wire was then placed at the spot where the wind was to be measured, and heated by electric current. It thus served to measure horizontal wind-velocity in any direction.

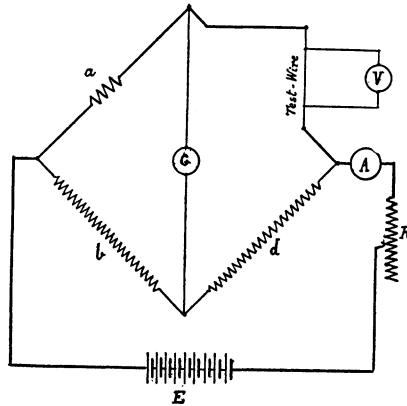


FIG. 12. Connections of Test-Wire for Indirect Measurements of Wind Velocity.

Two methods were used, one indirect-reading, the other direct-reading. In the indirect method, the connections were as shown in Fig. 12. Here the test-wire, shunted by a voltmeter, is placed in a Wheatstone bridge of unequal arms, so that the current in the test-wire side is ten times stronger than in the opposite side  $bd$ . The bridge is set for balance at a predetermined resistance and temperature of the test-wire. Whatever may be the horizontal velocity of the wind blowing over the test-wire, there is some current strength supplied to the bridge through a meter  $A$ , which will restore balance and zero current in the galvanometer  $G$ . When this balance is obtained, the readings of the ammeter  $A$ , and voltmeter  $V$ , are noted. Their product  $VA$ ; or the voltage square  $V^2$ , is proportional to the power dissipated in the 15 cm. of test-wire, from which the velocity of the wind can be deduced with the aid of suitably prepared tables or curves. The advantage of this method is its relatively high precision. Its disadvantages are that it requires to be adjusted for each observation, and in gusty winds, it is not possible to secure a Wheatstone-bridge balance long enough to obtain readings of either  $V$  or  $A$ .

In the direct-reading method, the connections are as shown in Fig. 13. Here the test-wire is connected across 110-volt lighting mains, through an adjusted resistance, which may consist of incandescent lamps, so as to receive as nearly a constant heating current as is practicable. The voltmeter  $V$  is connected to the potential taps, 15 cm. apart on the vertical test-wire. The apparatus is then set up at the place where the wind is to be measured. The four leads are

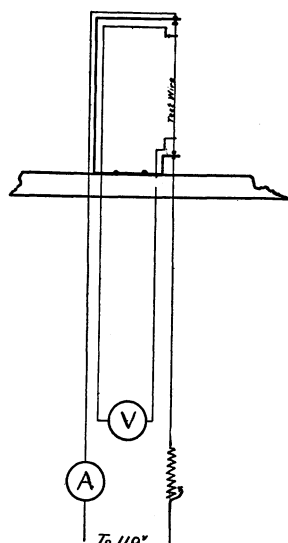


FIG. 13. Connections for Direct-Reading Type of Anemometer.

of any convenient length, and are bound up into a weatherproof cable. From calibration measurements made on a sample of the test-wire in a motor-driven fork, the linear convective dissipation of heat for any safe given linear resistance of the wire is known. As the horizontal component of wind-velocity increases, the temperature of the platinum test-wire falls, since no provision is made in this case to restore the initial temperature. A calibration curve has therefore to be prepared for a given exciting current, whereby the readings of the voltmeter, which may be a recording instrument, become convertible into wind-velocities. A set of calibration curves is given in Fig. 14 to the left-hand scale, both for 1.5 amperes and 2.0 amperes, constantly sup-

plied to the test-wire from the 110-volt circuit. It will be seen that when the wire is greatly cooled by the wind, a very appreciable correction for the temperature of the wind enters into the result; although at a temperature elevation of say  $300^{\circ}$  C, this correction would be comparatively small. It is necessary to set the current at such a value that when the wind fails, the test-wire shall not be dangerously heated. It was found that with a platinum test-wire of 0.114 mm. diameter, as used in these measurements, 1.5 amperes was a suitable current for wind-velocities up to 15 km. per hour. Thus, as indicated in Fig. 14, with a wind of 10 km. per hour, and a temperature of  $10^{\circ}$  C., the voltmeter reading was 4.3 volts. When, however, the wind-velocities were higher, the current was increased to 2 amperes, which, in still air, raised the wire temperature to visible redness. At 30 km. per hour, and  $10^{\circ}$  C. wind temperature, the p.d. on 15 cm. was then 5.8 volts.

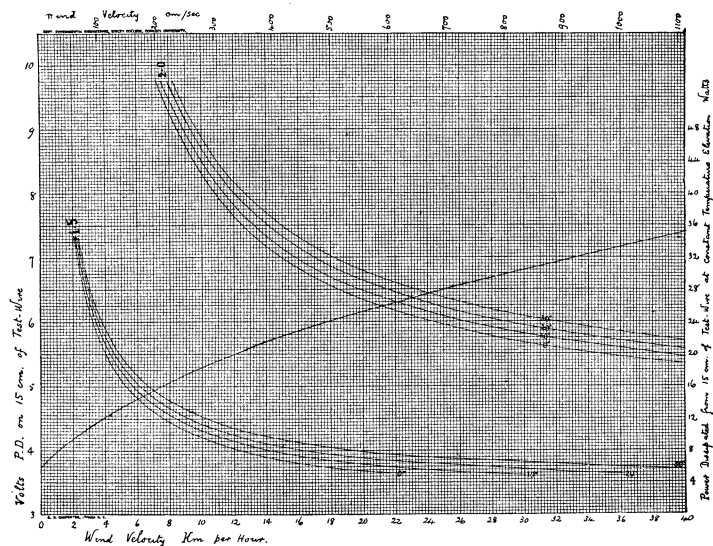


FIG. 14. Calibration Curves of 15 cm. Test Wire.

The G-support and test-wire were fastened to a pole and supported out of doors, exposed to the weather for some weeks. The apparatus appeared to be durable, and constant in its indications throughout that time, except that the solderings of the platinum tap-

wires across the test-wire, introduced an element of weakness. After exposure of some days, the test-wire was apt to break at a soldered point. In subsequent out-door trials, therefore, the tap-wires were simply twisted on to the test-wire. These tap-points involve a certain small error due to their cooling effect on the test-wire. It is, therefore, desirable to make the tap-wire as thin as is practicable, in view of both mechanical strength and electric resistance.

In a wind, the readings of the voltmeter anemometer are always fluctuating; but a mean value at any moment can always be estimated. A heavily damped voltmeter may conveniently be used.

The rising parabolic curve in Fig. 14 indicates, to the right-hand scale, the watts convected from 15 cm. of the same test-wire at different wind-velocities up to 1,100 cm./sec. or 40 km./hr. with constant resistance maintained in the wire; *i. e.*, by the method of Fig. 12. It is evident that for high wind-velocities, the indirect method of constant test-wire temperature is more sensitive than the direct-reading method of constant current. For lower velocities, however, the direct-reading method is much the more convenient, but requires to be corrected for the wind-temperature.

In the use of the constant-current direct-reading method, tungsten lamps have an advantage for the regulating resistance of Fig. 13 in that they tend to compensate for changes in the resistance of the test-wire at different wind-velocities; or to act as ballast resistance for the closer maintenance of constant current.

#### CONCLUSIONS.

1. The forced convection of heat from a thin platinum wire at constant temperature moved transversely through the air, varies not only approximately as the square root of the speed, but also approximately as the square root of the air-pressure (or air-density), in accordance with Boussinesq's formula.

2. At air-pressures below half an atmosphere, the above square-root relation was sensibly departed from. At pressures above 3 atmospheres, the relation was also departed from. Although the departures in both cases exceed the limits of observation errors, it is not certain whether they may have been due to imperfections in the

apparatus, such as eddy-currents in the confined air, caused by air-churning in the pressure-tank. In other words, it is uncertain whether, if a more spacious pressure-tank could have been used, with a longer fork radius, the square-root relation might not have held better than it did at the upper and lower air-pressures.

3. The effect of moisture in the air, upon the forced convection of heat from a thin wire, seems to be small, and has not yet been determined.

4. The forced convection of heat from a thin wire at constant temperature in air of variable pressure may be regarded as varying approximately with the square root of the mass of air displaced by the wire per second, or as  $\sqrt{v\sigma}$ , where  $v$  is the wind-velocity of displacement and  $\sigma$  the density of the air.

5. When the air-pressure  $p$  remains constant,  $(P_c/\theta)^2$  or the square of the linear convection per deg. C. elevation, plotted against wind-velocity  $v$ , gives a straight line for a thin metallic wire. When, however, the air-pressure  $p$  varies, then it seems that, at least to a first approximation,  $(P_c/\theta)^2/p$  plotted against wind-velocity  $v$ , gives one and the same straight line, for that wire, for all values of  $p$  from 0.5 to 2.0 megabars.

6. A thin vertical platinum wire about 25 cm. long, after being calibrated in a motor-driven fork, can serve as a recording wind measurer or anemometer. The record requires to be corrected both for the temperature and pressure of the air. The degree of precision obtainable is greatest at low wind-velocities.